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Effect of gas-phase and surface radiation on the structure and extinction of diffusion flames stabilized on a condensed fuel

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Abstract—The effect of gas-phase and surface radiation on the structure and extinction of a diffusion flame stabilized on a condensed fuel has been analyzed using a matched asymptotic expansion technique by adopting stagnation-point flow as the model problem. An assumption of an optically thin limit in the gas phase is applied in the analysis. Results show that extinction is caused by the surface radiation loss from a condensed fuel in the small-stretch regime while gas-phase radiation has a significant effect in the high-stretch regime. The dominant extinction mechanism for a diffusion flame on a condensed fuel is found to be fuel leakage in the high-stretch regime and both fuel and oxidizer leakage in the small-stretch regime.

INTRODUCTION

Heat loss and flame stretch are the two major causes of diffusion flame extinction. Heat loss reduces the flame temperature, thereby leading to flame extinction. The flow field effect manifested by flame stretch reduces the characteristic flow time such that there is not sufficient time available for a complete reaction. Therefore, if flame stretch increases to a certain level, flame extinction is expected.

The effect of radiation heat loss is frequently unaccounted for in flame structure analysis for two reasons. One is the difficulty associated with complex radiative transport in the analysis and the other is that the radiation effect is minimal for a laboratory scale flame. However, since real flames have a considerable size, especially diffusion flames of high carbon number fuels with a strong sooting tendency, the effect of radiation on flame characteristics needs to be considered. This radiation is an uncontrollable intrinsic heat loss mechanism [1] which affects various flame characteristics, such as flammability and extinction.

Adopting a matched asymptotic expansion technique, the effect of gas-phase radiative loss has been analyzed for diffusion flames in stagnation-point flow [2]. A numerical approach has been used to study the effect of surface radiation from a condensed fuel surface [3, 4], showing the importance of radiative extinction in the small-stretch regime; the asymptotic approach supports this finding [5]. The effect of radiation on premixed [6], droplet combustion [7] and stagnation-point flow [8] has also been investigated. An experimental study of stagnation-point flow with a condensed fuel geometry has been performed [9]. Recently, radiation was proposed as a possible mechanism for flammability [10].

In the present study, we have considered both the gas-phase and the surface radiation effect for a diffusion flame stabilized on a condensed fuel by adopting stagnation-point flow as the model problem. Using a matched asymptotic technique, the relative importance of these two modes of radiative loss has been identified.

GOVERNING EQUATIONS

With the assumptions of a one-step Arrhenius reaction, constant physical properties, the same mass diffusivities, and negligible viscous heating and pressure heating, the governing equations are as follows [11, 12]:

$$\frac{\partial(\rho u r^j)}{\partial r}+\frac{\partial(\rho v r^j)}{\partial y}=0$$

$$\rho u\frac{\partial u}{\partial r}+\rho v\frac{\partial u}{\partial y}-\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right)=\rho_e u_e\frac{\mathrm{d}u_e}{\mathrm{d}r}$$

$$\rho u\frac{\partial Y_i}{\partial r}+\rho v\frac{\partial Y_i}{\partial y}-\frac{\partial}{\partial y}\left(\rho D\frac{\partial Y_i}{\partial y}\right)=-\omega,\quad i=\mathrm{O},\mathrm{F}$$

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NOMENCLATURE

a flame stretch [s^{-1}]
a_o constant
B transfer number
b, b_w frequency factor [s^{-1}]
C_p, C_w specific heat for gas phase and solid
D mass diffusivity
Da Damköhler number
E_a, E_w activation energy of gas-phase and surface reactions
F_l nondimensional radiative heat loss
f stream function
G_l defined in equation (14)
g_i defined in equation (18)
h defined in equation (11)
j geometric factor
L nondimensional latent heat
l_p Plank mean absorption length
l_i radiative heat loss [equation (26)]
m burning rate
m_F fraction of heat transfer to fuel side
n_s local number density of soot particles
N_o stoichiometric oxidizer to fuel mass ratio
Q heat of combustion per unit mass of fuel consumed
Q_l nondimensional total radiative heat loss [equation (20)]
r, y coordinate
r_s radius of soot particle
R° universal gas constant
S ratio of surface radiation to convection
s similarity variable
T nondimensional temperature
T_a nondimensional activation temperature
u, v velocity
x transformed coordinate
Y scaled mass fraction
Y'_O oxidizer mass fraction
Y_O Y'_O/N_o
z transformed coordinate
Z transformed temperature function.

Greek symbols
α $Y_{O,e}$
β defined in equation (16)
γ temperature sensitivity factor [equation (18)]
Δ reduced Damköhler number
ε small parameter, T_f^2/T_a
ε_w surface emissivity
ζ stretched coordinate
η similarity variable
θ inner temperature
λ thermal conductivity
Λ radiative loss
μ viscosity
μ_o flame temperature loss parameter
ξ stretched coordinate
Θ inner coupling function
ρ density
σ Stefan–Boltzmann constant
ϕ inner temperature
ψ stream function
ω reaction rate.

Subscripts
E extinction
e oxidizer boundary
F fuel
f flame
i chemical species (i = O, F)
in inner
O oxidizer
out outer
s soot particles
w fuel surface.

Superscripts
+ outer nonreactive zone ($\xi_f < \xi$)
− outer nonreactive zone ($\xi < \xi_f$).

$$\rho u\frac{\partial T}{\partial r}+\rho v\frac{\partial T}{\partial y}-\frac{\partial}{\partial y}\left(\frac{\lambda}{C_p}\frac{\partial T}{\partial y}\right)+\frac{4\sigma T^4 Q^3}{l_p C_p^4}=\omega$$

$$\omega = -Da\, Y_F Y_O \exp(-T_a/T) \quad (1)$$

where u, v and r, y are the radial and axial velocities and coordinates, respectively, ρ the density, μ the viscosity, D the mass diffusivity, λ the thermal conductivity, T and T_a are the temperature and activation temperature (E_a/R°) nondimensionalized by the characteristic temperature Q/C_p, Q the heat of combustion, C_p the specific heat, R° the universal gas constant, Y_F the fuel mass fraction, Y_O the oxidizer mass fraction Y'_O divided by the stoichiometric oxidizer to fuel mass ratio N_o, Da the Damköhler number defined as $bN_o/[a(j+1)]$, b the frequency factor, a the flame stretch, and j the geometric factor which is 0 for the two-dimensional case or 1 the axisymmetric case.

Here, the gas-phase radiation term in the energy equation is written on the basis of an optically thin gas approximation such that the radiation flux can be modeled as

$$E_R''' = 4\sigma T^4/l_p \quad (2)$$

where σ is the Stefan–Boltzmann constant, and l_p the

Plank mean absorption length. If the radiation loss from soot particles is the dominant factor, then l_p can be modeled as [1, 2]

$$l_p = (T/T_s)^4/(\pi r_s^2 n_s) \tag{3}$$

where n_s is the local number density, T_s the surface temperature, and r_s the radius of spherical soot particles having a surface emissivity of unity. Under this assumption, radiative heat loss on the oxidizer side can be neglected since soot particles are usually located on the fuel side of the flame. Here, l_p decreases with T since $n_s r_s^2$ increases rapidly with the increase in T due to the chemical kinetics of soot production and particle growth [2].

The boundary conditions are as follows:

$$y \to \infty\,;\, u = u_e \quad T = T_e \quad Y_F = 0 \quad Y_O = Y_{O,e}$$

$$y = 0\,;\, u = 0$$

$$\rho v Y_O - \rho D \frac{\partial Y_O}{\partial y} = 0 \quad \rho v Y_F - \rho D \frac{\partial Y_F}{\partial y} = \rho v$$

$$\frac{\partial T}{\partial y} = \frac{\rho v}{\lambda/C_p}(T_w - T_e C_w/C_p) + \frac{\rho v}{\lambda/C_p} L + \frac{\varepsilon_w \sigma}{\lambda} \frac{Q^3}{C_p^3} T_w^4 \tag{4}$$

where C_w is the specific heat, L is the latent heat nondimensionalized by the heat of combustion, and ε_w the surface emissivity of the condensed fuel. The boundary conditions at $y = 0$ imply the impermeability of the oxidizer and a convection and diffusion balance with vaporization of fuel. The thermal boundary condition indicates that the heat conduction from the gas phase is equal to the semi-infinite one-dimensional heat conduction inside the condensed fuel having a temperature of T_e at $y \to -\infty$ plus the heat of vaporization and surface radiation.

The relation of the vaporization rate to the surface temperature is assumed in the Arrhenius form as

$$\rho v = b_w \exp(-T_{a,w}/T_w) \tag{5}$$

where b_w is the preexponential factor, and $T_{a,w}$ the activation temperature of the condensed fuel.

Adopting the similarity variables η and s, and the stream function ψ:

$$s = \int_0^r \rho_e \mu_e u_e r^{2j}\,\mathrm{d}r \quad \eta = \frac{u_e r^j}{(2s)^{1/2}} \int_0^y \rho\,\mathrm{d}y$$

$$\psi = (2s)^{1/2} f(\eta)$$

$$\frac{\partial \psi}{\partial y} = \rho u r^j \quad -\frac{\partial \psi}{\partial r} = \rho v r^j \tag{6}$$

and assuming unity Prandtl (Pr) and Lewis (Le_i) numbers and constant $\rho\mu$, equation (1) can be transformed into

$$\frac{\mathrm{d}^3 f}{\mathrm{d}\eta^3} + f\frac{\mathrm{d}^2 f}{\mathrm{d}\eta^2} - \frac{1}{j+1}\left\{\left(\frac{\mathrm{d}f}{\mathrm{d}\eta}\right)^2 - 1\right\}$$

$$\frac{\mathrm{d}^2 Y_i}{\mathrm{d}\eta^2} + f\frac{\mathrm{d}Y_i}{\mathrm{d}\eta} = \omega \quad i = \mathrm{F,O}$$

$$\frac{\mathrm{d}^2 T}{\mathrm{d}\eta^2} + f\frac{\mathrm{d}T}{\mathrm{d}\eta} = -\omega + F_l(T)$$

$$F_l(T) = \frac{4\sigma T^4}{l_p Q} \frac{1}{2^j \rho a} \tag{7}$$

with the boundary conditions

$$\eta \to \infty\,;\, f' = 1 \quad T = T_e \quad Y_F = 0 \quad Y_O = Y_{O,e}$$

$$\eta = 0\,;\, f' = 0 \quad \frac{\mathrm{d}Y_O}{\mathrm{d}\eta} = -f_w Y_{O,w} \quad \frac{\mathrm{d}Y_F}{\mathrm{d}\eta} = f_w(1 - Y_{F,w})$$

$$\frac{\mathrm{d}T}{\mathrm{d}\eta} = -f_w[(T_w - T_e C_w/C_p) + L] + \frac{\varepsilon_w \sigma Q^3}{C_p^4 \sqrt{j+1}} \frac{T_w^4}{\sqrt{\rho_e \mu_e a}} \tag{8}$$

where the vaporization rate is

$$m = (j+1)^{1/2}(\rho_e \mu_e a)^{1/2}(-f_w). \tag{9}$$

Thus, equation (5) can be expressed as follows:

$$-f_w = \frac{1}{\sqrt{j+1}} \frac{b_w}{\sqrt{\rho_e \mu_e a}} \exp(-T_{a,w}/T_w). \tag{10}$$

In equation (7) the energy and species equations are related to the convection term only through f and the boundary condition through f_w. If the momentum equation in terms of f_w is found then the species and energy equations can be analyzed separately.

By introducing a new variable x as

$$x = -\frac{f_w}{(1+B)} \int_\eta^\infty e^{-h}\,\mathrm{d}\eta \quad h = \int_0^\eta f\,\mathrm{d}\eta \tag{11}$$

equations (7) and (8) can be transformed into

$$\frac{\mathrm{d}^2 Y_i}{\mathrm{d}x^2} = \left(\frac{1+B}{f_w}\right)^2 e^{2h}\omega \quad i = \mathrm{F,O}$$

$$\frac{\mathrm{d}^2 T}{\mathrm{d}x^2} = -\left(\frac{1+B}{f_w}\right)^2 e^{2h}[\omega + F_l(T)] \tag{12}$$

$$x = 0\,;\, T = T_e \quad Y_F = 0 \quad Y_O = Y_{O,e}$$

$$x = x_w\,;$$

$$\frac{\mathrm{d}Y_O}{\mathrm{d}x} = -(1+B)Y_{O,w} \quad \frac{\mathrm{d}Y_F}{\mathrm{d}x} = (1+B)(1 - Y_{F,w})$$

$$\frac{\mathrm{d}T}{\mathrm{d}x} = \frac{(1+B)}{f_w}[(-f_w)(T_w - T_e C_w/C_p + L) + ST_w^4] \tag{13}$$

where S is the ratio of surface radiation to convection loss, and B is the transfer number, defined, respectively, as

$$S = \frac{\varepsilon_w \sigma Q^3}{C_p^4 \sqrt{j+1}} \frac{1}{\sqrt{\rho_e \mu_e a}}$$

$$B = \frac{\alpha + T_e - T_w - G_{l,w} - Y_{O,w}}{T_w - T_e C_w/C_p + L - ST_w^4/f_w}$$

$$G_l = -\int_0^x \int_0^\eta (d\eta/dx) F_l(T)\, d\eta\, dx \qquad (14)$$

with $G_{l,e} = 0$ and $G'_{l,w} = 0$, and the fuel surface becoming $x_w = B/(1+B)$.

ANALYSIS

The reaction and gas-phase radiation are temperature-sensitive, so that these effects will be confined to a relatively thin zone near the higher-temperature region, outside of which there are nonreactive nonradiative regions. Thus, the flow field is divided into an outer zone, a radiative zone, and a reaction zone.

Outer zone

Using equations (12) and (14), the Shvab–Zeldovich coupling function becomes

$$\frac{d^2(T + Y_i + G_l)}{dx^2} = 0 \quad i = F, O. \qquad (15)$$

This has the following flame sheet solution using the boundary conditions of equation (13):

$$T + Y_F = T_e - G_l + (1 - \beta + G_{l,w})x$$

$$T + Y_O = T_e + \alpha - G_l - (\alpha + \beta - G_{l,w})x$$

$$\beta = [T_w - T_e C_w/C_p + L - ST_w^4/f_w] + T_e - T_w \qquad (16)$$

from which the flame location can be determined by the assumption of complete consumption of fuel and oxidizer at the flame as $x_f = \alpha/(1+\alpha)$, where $\alpha = Y_{O,e}$.

In the Burke–Schumann limit of $Da \to \infty$, equation (12) can be rewritten as

$$\frac{d^2 Y_i}{dx^2} = (1+\alpha)\delta(x - x_f) \quad i = O, F$$

$$\frac{d^2 T}{dx^2} = \left(\frac{1+B}{f_w}\right)^2 e^{2h} F_l(T) - (1+\alpha)\delta(x - x_f). \qquad (17)$$

Adopting a similar approach to ref. [2], the temperature sensitivity of the gas-phase radiation loss can be defined as $\gamma_i = d \ln F_{l,i}(T)/d \ln T$; if γ_i is constant $F_{l,i}(T) \propto T^{\gamma_i}$. Here, the temperature sensitivity on the fuel and oxidizer sides can be treated separately, allowing for the different radiation characteristics of soot formation. Assuming γ_i to be large and letting $g_i = d \ln \gamma_i(T)/d \ln T$, and the temperature stretch $\phi = \gamma_{i,f}(T_f - T)/T_f$, the radiative loss term can be expanded as

$$F_l(T) = F_{l,f} e^{-\phi}[1 + (g_{i,f} - 1)\phi^2 \gamma_{i,f}^{-1} + \ldots] \qquad (18)$$

where the subscript f denotes the value at the flame position.

If the gas-phase radiation is sensitive to temperature, it will be confined to the vicinity of the maximum-temperature region. Thus, in the outer nonreactive and nonradiative zones, the temperature profiles can be expressed as a linear function of x from equation (17)

$$T_{out}^- = T_e + (T_f - T_e)x/x_f \quad x < x_f$$

$$T_{out}^+ = T_e - \beta + (T_f - T_e + \beta)(1-x)/(1-x_f) \quad x > x_f. \qquad (19)$$

The total gas-phase radiation loss can be determined from the integration of equation (17) as

$$Q_l = (1+\alpha)(1-\beta)(1-\mu_o) \qquad (20)$$

where the flame temperature loss parameter μ_o (ranging from 0 to 1) is

$$\mu_o = (T_f - T_e)/(T_{f,ad} - T_e) \qquad (21)$$

where $T_{f,ad}$ is the adiabatic flame temperature when there is no radiative loss.

Radiation zone

As a first approximation, dG_l/dx can be neglected on the oxidizer side of $x < x_f$. Using $G_{l,e} = 0$ then, from equation (17), one finds that

$$G_{l,w} = Q_l \alpha/(1+\alpha). \qquad (22)$$

Thus, equation (17) can be rewritten as

$$\frac{d^2 T}{dx^2} = [Q_l - (1+\alpha)]\delta(x - x_f). \qquad (23)$$

In analyzing the radiative zone, the following stretched variable is introduced:

$$\xi = \gamma_{i,f}(x - x_f)/T_f. \qquad (24)$$

Then, by using equation (18), equation (17) can be transformed into

$$\frac{d^2\phi}{d\xi^2} = (1+\alpha)\delta(\xi) - l_i e^{-\phi} \qquad (25)$$

where the ratio of radiant energy emission to enthalpy convection l_i becomes

$$l_i = \left(\frac{1+B}{f_w}\right)^2 e^{2h_f} \frac{T_f}{\gamma_{i,f}} F_{l,f} \quad i = O, F. \qquad (26)$$

The matching conditions for the outer solution of equation (19) are

$$\left.\frac{d\phi}{d\xi}\right|_\infty = (T_f - T_e + \beta)(1+\alpha)$$

$$\left.\frac{d\phi}{d\xi}\right|_{-\infty} = -(T_f - T_e)(1+\alpha)/\alpha. \qquad (27)$$

Hence, integrating equation (25) away from $\xi = 0$ and evaluating constants at $\xi = 0$ using equation (24), one finds that

$$\left.\frac{d\phi}{d\xi}\right|_{0^+} = [2l_F+(T_f-T_e-\beta)^2(1+\alpha)^2]^{1/2}$$

$$\left.\frac{d\phi}{d\xi}\right|_{0^-} = -[2l_O+(T_f-T_e)^2\{(1+\alpha)/\alpha\}^2]^{1/2}. \quad (28)$$

Therefore, the flame temperature loss can be found as

$$1+\alpha = [2l_O+\mu_O^2(1-\beta)^2]^{1/2}+[2l_F+\mu_F^2(\alpha+\beta)^2]^{1/2} \quad (29)$$

where

$$\mu_F = (T_f-T_e+\beta)(1+\alpha)/(\alpha+\beta)$$
$$= (T_f-T_e+\beta)/(T_{f,ad}-T_e+\beta). \quad (30)$$

For diffusion flames, $l_O \ll l_F$ due to the location of soot production and the consumption zone, which is usually located on the fuel side of a flame; thus, the flame temperature can be derived by neglecting l_O as

$$T_f = T_{f,ad}-\frac{\alpha(\alpha+\beta)}{(1-\alpha^2)}\left[\left\{1+\frac{2l_F(1-\alpha)}{(1+\alpha)(\alpha+\beta)^2}\right\}^{1/2}-1\right]. \quad (31)$$

Reaction zone

If the ratio of the thickness of the reaction zone $O(T_f^2/T_a)$ to that of the radiative loss zone $O(T_f/\gamma_{i,f})$ is small, then the chemical reaction is imbedded in the radiation zone, while, if it is of the same order, then radiation and reaction occur simultaneously in the same region. Here, we will start with the general latter case and later specialize to the former case. Assuming the ratio $a_o = \gamma_{i,F}T_f/T_a = O(1)$ and expanding the coordinate and independent variables as

$$\zeta = (x-x_f)(1+\alpha)(T_a/T_f^2)$$
$$\theta = (T_f-T)(T_a/T_f^2)$$
$$\Theta = (Y_O+T-T_f)(T_a/T_f^2) \quad (32)$$

then

$$Y_O = (\Theta+\theta)(T_f^2/T_a)$$
$$Y_F = (\zeta+\Theta+\theta)(T_f^2/T_a). \quad (33)$$

Substituting these into equation (12), the inner governing equations become

$$\frac{d^2\Theta}{d\zeta^2} = \Lambda\exp(-a_o\theta)$$

$$\frac{d^2\theta}{d\zeta^2} = \Delta(\Theta+\theta+\zeta)(\Theta+\theta)\exp(-\theta)-\Lambda\exp(-a_o\theta) \quad (34)$$

where

$$\Lambda = \left(\frac{1+B}{f_w}\right)^2\exp(2h_f)\frac{T_f^2}{T_a}\frac{F_{l,f}}{(1+\alpha)^2}$$

$$\Delta = \left(\frac{1+B}{f_w}\right)^2\exp(2h_f)\left(\frac{T_f^2}{T_a}\right)^3\frac{1}{(1+\alpha)^2}Da\exp(-T_a/T_f) \quad (35)$$

are the radiation loss parameter and the system Damköhler number, respectively. The boundary conditions are

$$\zeta\to-\infty: \frac{d\theta}{d\zeta} = -\frac{d\Theta}{d\zeta}\to T_f-T_e+\beta$$

$$\zeta\to\infty: -\frac{d\theta}{d\zeta} = 1+\frac{d\Theta}{d\zeta}\to(T_f-T_e)/\alpha. \quad (36)$$

In general, equation (34) and equation (36) can be solved numerically. However, if Λ is so small as to be of the order of (T_f^2/T_a), and, if a_o also is small, then l_i becomes negligible in this zone; hence, the term involving Λ in equation (34) can be neglected.

If the reaction zone is much thinner than the gas-phase radiative zone, equation (27) can be used as the outer boundaries of equation (34), which are

$$\zeta\to-\infty: \frac{d\theta}{d\zeta} = m_F-1$$

$$\zeta\to\infty: \frac{d\theta}{d\zeta} = m_F \quad (37)$$

where m_F is the fraction of heat transfer to the fuel surface, written as

$$m_F = [2l_F/(1+\alpha)^2+(T_f-T_e+\beta)^2]. \quad (38)$$

Under these assumptions, the solution for Θ from equation (34) becomes $\Theta = -m_F\zeta$ as a first approximation. Substituting into θ and letting

$$z = (4\Delta)^{1/3}\zeta/2$$
$$Z = (4\Delta)^{1/3}[\theta+(1-2m_F)\zeta/2] \quad (39)$$

the governing equation and boundary conditions become

$$\frac{d^2Z}{dz^2} = (Z-z)(Z+z)\exp\{-(4\Delta)^{-1/3}[Z+(2m_f-1)z]\}$$

$$\zeta\to-\infty: \frac{dZ}{dz} = -1 \quad \zeta\to\infty: \frac{dZ}{dz} = 1. \quad (40)$$

The extinction criterion for these systems is given in ref. [13] as

$$\Delta_E = ec\{1-2c+1.04c^2+0.44c^3\}/2$$
$$c = \begin{cases} m_F & m_F \leqslant 1/2 \\ 1-m_F & m_F \geqslant 1/2 \end{cases} \quad (41)$$

and extinction is expected if $\Delta < \Delta_E$.

RESULTS AND DISCUSSIONS

To identify the effect of gas-phase radiation and the surface radiation from a condensed fuel surface, the system and extinction Damköhler numbers of equa-

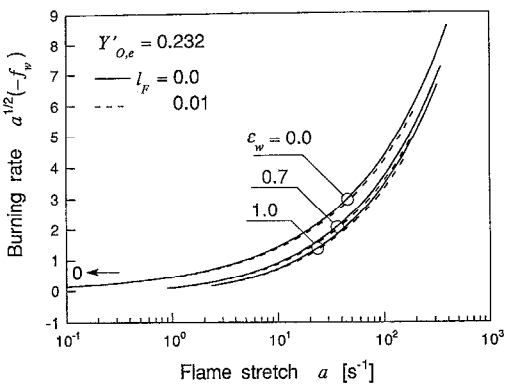

Fig. 1. Effect of flame stretch on burning rate of condensed fuel.

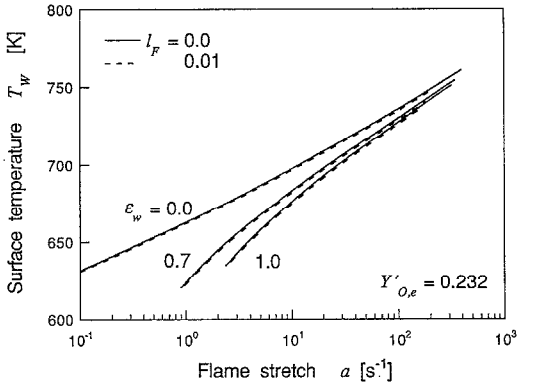

Fig. 2. Effect of flame stretch on surface temperature.

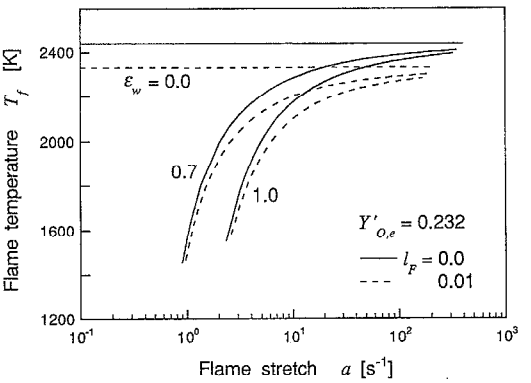

Fig. 3. Effect of flame stretch on flame temperature.

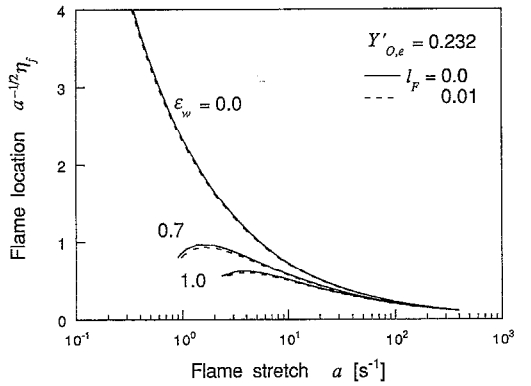

Fig. 4. Effect of flame stretch on flame location.

tions (35) and (41) should be evaluated. Since the energy and species equations are related to the momentum equation through f_w, the momentum equation should also be solved which is, in general, coupled with the energy and species equations. However, to identify the relative importance of radiation on flame characteristics and extinction, an incompressible assumption can be used. Since both radiation and reaction occur in a relatively thin zone where diffusion is balanced with these effects, the dominant effect can be identified even with an incompressible assumption, as was frequently adopted in the literature [12, 14]. In the incompressible assumption, if the characteristics of f are given *a priori*, the calculation is much simplified. It has been found that the following relations can be represented as an approximate solution to the momentum equation for the axisymmetric case [5]:

$$f(\eta) = \{1/f''(0)\} \ln \{\cosh f''(0)\eta\} + f_w$$
$$f''(0) = a_1 + a_2 f_w : a_1 = 0.91769 \quad a_2 = 0.5099. \tag{42}$$

Since gas-phase radiation is mainly from soot particles for diffusion flames, only the contribution from the fuel side has been considered ($l_O = 0$). The optically thin approximation implies that all the radiation from the gas phase and the surface radiation are lost to the ambient environment without any self-absorption. A typical laboratory flame has the gas-phase radiation characteristics $F_{l,f} = O(10^{-6})$; however, to demonstrate the effect of gas-phase radiation, a large gas-phase radiation parameter has been used in the calculation. The temperature sensitivity is assumed to be $\gamma_{F,f} = 5$. The condensed fuel modeled is polymethylmethacrylate and the physico-chemical parameters have been adopted from refs. [3, 4].

By fixing the oxidizer concentration as $Y'_{O,e} = 0.232$, various flame characteristics are calculated as a function of the flame stretch, which are shown in Figs. 1–5, where the end points of the curves indicate the extinction satisfying the criterion $\Delta < \Delta_E$.

First, consider the case of neglecting gas-phase radiation ($l_F = 0$). If the surface radiation is also negligible ($\varepsilon_w = 0$), the flame temperature and fraction of heat transfer to the fuel surface from the flame remain unchanged with the increase in flame stretch, while the surface temperature and burning rate $\{a^{1/2}(-f_w)\}$ increase, resulting from the decrease in the distance of the flame from the fuel surface ($a^{-1/2}\eta_f$). In this case flame extinction is only possible by increasing the stretch.

With the surface radiation from the condensed fuel surface, the burning rate and surface temperature characteristics with stretch are similar to that without surface radiation in the high-stretch regime with a

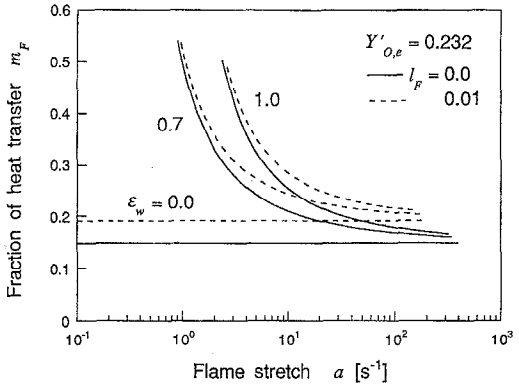

Fig. 5. Effect of flame stretch on fraction of heat transfer to fuel side.

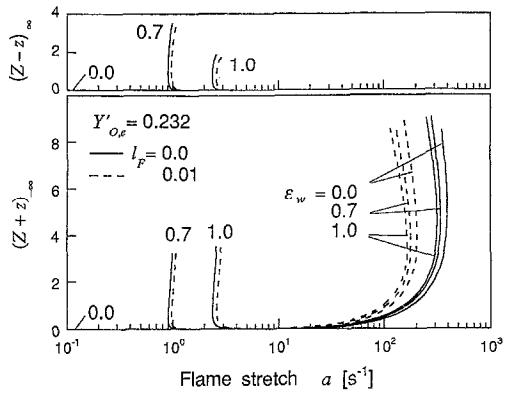

Fig. 6. Effect of flame stretch on reactant leakages.

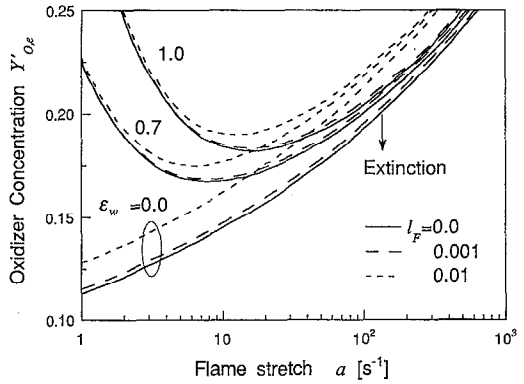

Fig. 7. Flammable boundary in oxidizer concentration vs flame stretch domain.

small decrease in the extinction stretch values. However, as the stretch decreases, the flame temperature decreases rapidly and the fraction of heat transfer increases as shown in Figs. 3 and 5. The flame location is closer to the surface as ε_w increases, as can be seen in Figs. 4 and 5; thus, a large amount of heat is transferred to the wall resulting in flame extinction. Therefore, with surface radiation, both high- and low-stretch limits exist. In such a case the fraction of heat transfer reaches approximately 0.5 in the low-stretch extinction limit. In general, the extinction stretch decreases with the increase in surface radiation in the high-stretch limit and increases in the low-stretch limit. The high-stretch limit is less sensitive than the low-stretch limit to surface radiation.

For gas-phase radiation, the flame temperature decreases while the fraction of heat transfer to the fuel side increases considerably, even if there is no surface radiation. Other characteristics such as the burning rate, the surface temperature, and the flame location have changed minimally, except that the high-stretch extinction limit has been significantly reduced.

For the case of both surface and gas-phase radiation the gas-phase radiation is insignificant in the low-stretch regime while the flame temperature, fraction of heat transfer, and extinction are significantly affected in the high-stretch regime. The reason for this is that in the low-stretch regime the flame is too far away from the fuel surface for gas-phase radiation to affect various flame characteristics.

To identify the extinction mechanism, the fuel $\{(Z+z)_{-\infty}\}$ and oxidizer $\{(Z-z)_{\infty}\}$ leakages are plotted in Fig. 6. If there is no surface radiation, fuel leakage leads to extinction as the stretch increases. If there is surface radiation, high-stretch extinction is due to fuel leakage while the low-stretch limit is due to both fuel and oxidizer leakage. This is because the fraction of heat transfer approaches 0.5 as the stretch decreases, meaning that both sides of the flame have comparable temperature gradients. Therefore, as the stretch decreases, the structure of the flame changes from a nearly frozen oxidizer side and near-equilibrium fuel side for small m_F to the both sides being nearly frozen for m_F close to 0.5.

The flammable boundary resulting from varying the oxidizer concentration in the free stream is shown in Fig. 7. As mentioned previously, surface radiation has a significant effect in the low-stretch regime while the gas-phase effect is dominant in the high-stretch regime.

CONCLUDING REMARKS

The relative importance of gas-phase and surface radiation for diffusion flames stabilized on a condensed fuel has been analyzed using a matched asymptotic expansion technique. Surface radiation is the dominant mechanism for flame extinction in the low-stretch regime while the gas-phase radiation effect is minimal. In the high-stretch regime, gas-phase radiation has a significant effect on the stretch at flame extinction and the effect of surface radiation is negligible in this regime. The flammable range of oxidizer concentration vs stretch agrees well with the numerical studies [3, 4] considering only the surface radiation effect. the extinction mechanism in the low-stretch regime is found to be caused by both fuel and oxidizer leakage through the flame, while it is caused by fuel leakage alone in the high-stretch regime.

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